STRUCTURE OF ANODE SHEATH IN SELF-SUSTAINING DISCHARGE WITH CLOSED ELECTRON DRIFT

V. K. Kalashnikov and Yu. V. Sanochkin

UDC 533.9.03

The structure of the anode sheath and the characteristics of the closed Hall current discharge occurring in plasma accelerators are investigated in the diffusion approximation. The analysis takes account of "weak" processes (variation of ionization frequency with electron energy, electron energy losses, presence of a detachment region at the anode). The distributions along the sheath of potential, particle densities, currents, and other parameters are obtained. The effect on the characteristics of the discharge of varying the magnetic field and the input of working substance is considered. Weak processes are shown to have a significant effect on the structure of the sheath.

Discharges in a strong transverse magnetic field (which are of importance in plasma accelerators, in obtaining fluxes of high-energy ions, and so on) have been intensively studied in recent times (see references in [1]). The discharge in such devices for a typical discharge voltage φ_0 on the kilovolt level and a magnetic field $H \sim 1$ kOe is characterized by the formation of a positive anode potential fall almost equal to φ_0 in a sheath of thickness on the order of the Larmor radius of electrons of energy $e\varphi_0$.

This sort of discharge is observed to occur in various identifiable modes. At pressures $p < 10^{-5}$-10^{-4} torr (1 torr = 133.322 Pa) there is observed a low-pressure discharge (the "vacuum mode") characterized by a small probability of ionization of neutrals in the sheath ($P \ll 1$), by a discharge current density of not more than a few mA/cm^2, by an ionic space-charge density negligible compared with the electronic, and so on [2, 3]. When the pressure (or flow rate) is increased above a certain critical value ($p > 10^{-3}$ -10^{-2} torr), the discharge goes over abruptly into a "heavy-current" mode with intense ion formation (mode of efficient generation of ions, $P \simeq 1$) [4] and quasineutral in the greater part of the anode sheath. At intermediate pressures (flow rates) a discharge mode occurs which may formally be designated as the "transition" mode. In this case, unlike the vacuum mode, the neutral density is appreciably nonuniform along the sheath ($0 < P < 1$), while the densities of ions and electrons may be comparable in magnitude. The structure of the anode sheath of a self-sustaining discharge was solved in [5, 6] for the vacuum and transition modes. To simplify the analysis a number of effects were disregarded: dependence of ionization frequency on electron energy in the sheath $\nu_i(W)$, electron energy losses in collisions $\mathscr{E}_i$ the presence at the anode of a detachment region of thickness d depleted of electrons, and so on. It will be seen below that each of these processes can be put into correspondence with a small parameter and in this sense can be formally designated as "weak." It is shown in [3], however, that in the low-pressure mode the weak processes have a significant effect on the characteristics of the discharge.

We consider the effect of the weak processes specified above on the structure of the anode sheath with increasing pressure (flow rate) and the change in the characteristics of the discharge with varying magnetic field. The heating of the electron gas in the discharge is of interest in its own right.

The sheath model employed in [5, 6] is incorrect in the sense that it leads to a solution that does not permit of fitting to the quasineutral solution in the plasma. This is a consequence of using null boundary conditions for the electric field E_0 and the electron flux j_0 at the beginning of the sheath. In actuality, these quantities have certain finite values that are subject to definite relationships analogous to the well-known

Kaliningrad, Moscow Region. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 9-16, March-April, 1976. Original article submitted June 26, 1975.

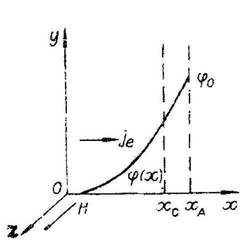

Fig. 1

Bohm criterion [7]. Accordingly, equality of the ion and electron densities at the beginning of the sheath is utilized in the set of boundary conditions in the present paper.

We consider the planar problem (as in [3, 5]). The x axis is directed across the sheath towards the anode; the z axis is along the external uniform magnetic field (Fig. 1). The origin of coordinates is located at some point where the electric field and the energy of the electrons are small (the beginning of the sheath). The system is uniform in the directions y and z. The Hall current is directed along the y axis and closes at infinity. A detachment region lies between the diffusion part of the sheath and the anode, i.e., at x in the range $x_d < x \leq x_A$. If, in a closed Hall current accelerator, the flux of neutrals enters the discharge through the anode, then the sought quantities are distributed in the diffusion part of the sheath in accordance with the equations

$$dj_e/dx = dq/dx = \nu_i n_e \; (\nu_i = \langle \sigma_{ie} v_e \rangle n_n);$$
$$j_e = b_\perp n_e d\varphi/dx \; (b_\perp = (e/m)(\nu_0/\omega_e^2), \; \nu_0 = \tau^{-1} = \langle \sigma_0 v_e \rangle n_n),$$
$$(d/dx)(j_e W) = -e j_e E - \mathscr{E}_i \nu_i n_e; \; dE/dx = -4\pi e(n_e - n_i), \; E = -d\varphi/dx; \tag{1}$$
$$n_i = \int_x^{x_d} \frac{\nu_i n_e dx'}{\sqrt{\frac{2e}{M}(\varphi' - \varphi)}}; \; \langle \sigma_{ie} v_e \rangle = \langle \sigma_{ie} v_e \rangle_0 \, \nu(W/eV_i),$$
$$\nu = (1/0.57)\sqrt{eV_i/W} \cdot Ei(W/eV_i), \langle \sigma_{ie} v_e \rangle_0 = (1.14/\sqrt{\pi})\sqrt{2eV_i/m} \cdot \sigma_{i0},$$

where j_e and q are the flux densities of electrons and neutrals; n_e, n_i, n_n are the densities of electrons, ions, and neutrals, respectively; φ is the potential; ν_i is the frequency of electron ionizing collisions; $b_\perp$ is the transverse electron mobility; $\sigma_{i,e}$ and σ_0 are, respectively, the cross section for ionizing collisions and the total cross section for collisions with loss of electron directional momentum; σ_{i0} is the characteristic cross section for the given substance; v_e is the thermal electron velocity; V_i is the ionization potential of the working substance; $\mathscr{E}_i$ is the effective ion value; W is the mean electron energy in cross section x; Ei (x) is the exponential integral [8]; m and M are the electron and ion masses; and ω_e is the electron cyclotron frequency.

In (1) we neglect the gradient of the electron pressure compared with the mobility. This is given a basis in [2, 10], where it is shown that the dominant mechanism responsible for electron transport to the anode up to pressures $\sim 10^{-3}$ torr is classical transverse mobility. The results of calculations on the heating and the density of the electrons in the sheath also indicate that diffusion transport of electrons does not play any significant role under the investigated conditions. The energy expended on a single act of ionization includes elastic and inelastic losses and depends on W. For the aims set in the present work we can set $\mathscr{E}_i$ = const of the order of a few eV_i. The lifetime of electrons in the detachment region is less than in the diffusion part of the sheath by a factor of $\omega_e \tau_e$. Accordingly, in order to assess the affect of the detachment region on the discharge parameters, we may utilize a rough sheath model in which the smooth variation of n_e in the vicinity of x_d is disregarded and in which n_e is set equal to zero within the detachment region. At the anode we then have

$$E(x_A) = E(x_d); \; j_e(x_A) = j_e(x_d); \; W(x_A) = W(x_d) + d_d E(x_d);$$
$$\varphi(x_A) = \varphi_0 = \varphi(x_d) + d_d E(x_d) \; (d_d = x_A - x_d). \tag{2}$$

[The latter of equalities (2) can be used to determine x_d, the beginning of the detachment region.] The width of the detachment region averaged over electron velocities equals, correct to a factor on the order of unity, the electron Larmor radius in cross section x_d [3]:

$$d_d = (1/\omega_e)\sqrt{(\pi/2_m) W(x_d)}$$

with boundary conditions

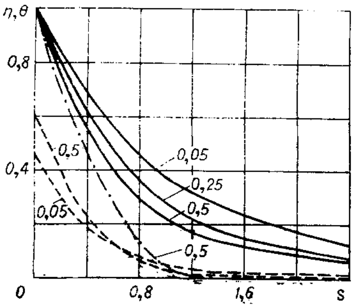

Fig. 2

$$\varphi(0) = 0;\quad E(0) = E_0;\quad W(0) = W_0;$$
$$n_e(0) = n_i(0);\quad q(x_d) = q_0 = n_0 v_0,$$

where n_0 is the density of neutrals at the anode and v_0 is their mean directional velocity. There is a certain arbitrariness in the choice of W_0, j_0, and E_0. Physical considerations indicate that it is reasonable to prescribe W_0 at the level of 1 eV; one of the boundary values j_0 or E_0 can be found from the condition of quasineutrality, but the other remains indeterminate. However, as was shown in [3] and confirmed by control calculations, varying j_0, E_0, and W_0 over wide limits has scarcely any effect on the characteristics of the sheath with the exception of its formal length x_A. We introduce the following scales and dimensionless quantities (the primes will be omitted in the future):

$$s = x/l^*;\ \eta = \varphi/\varphi_0;\ n_e' = n_e/n^*;\ n_i' = n_i/n^*;$$
$$j_e' = j_e/q^*;\ q' = q/q^*;\ \Theta = W/e\varphi_0;\ \rho_d = d_d/l^*;$$
$$l^* = \left[(e\varphi_0/m\omega_e^2)\left(\langle\sigma_0 v_e\rangle/\langle\sigma_{ie} v_e\rangle\right)\right]^{1/2},\ n^* = v_0/\langle\sigma_{ie} v_e\rangle l^*,\ q^* = q_0.$$

The sheath parameters vary with distance in accordance with the boundary problem:

$$dq/ds = dj/ds = \nu q n_e,\quad \nu = (1/0.57)(1/\sqrt{k\Theta})\,Ei(k\Theta),$$
$$j = q n_e d\eta/ds,\quad (d/ds)(j\Theta) = jE - \mathscr{E}\nu q n_e;$$
$$\varkappa d^2\eta/ds^2 = n_e - \beta\int_s^{s_d}\frac{\nu' q' n_e' ds'}{\sqrt{\eta' - \eta}}; \tag{3}$$
$$E(0) = E_0;\quad \Theta(0) = \Theta_0;\quad \eta(0) = 0;\quad n_e(0) = n_i(0) = j_0/q_0E_0; \tag{4}$$
$$q(s_d) = 1;\quad \eta_d = 1 - \rho_d E_d,\quad \text{where}$$
$$\beta = \sqrt{(M/2m)\left(\nu_i\nu_0/\omega_e^2\right)};\ \varkappa = (m\omega_e/4\pi e^2)\left(e\varphi_0/mv_0^2\right)^{1/2}\cdot\langle\sigma_{ie}v_e\rangle^{3/2}/\langle\sigma_0 v_e\rangle^{1/2}; \tag{5}$$
$$k = \varphi_0/V_i;\ \mathscr{E} = \mathscr{E}_i/e\varphi_0.$$

The problem contains four dimensionless criteria (5). The parameters k and $\mathscr{E}$ for a fixed φ_0 are determined primarily by the properties of the working substance. By (5), $\beta \sim n_0/H$ and $\varkappa \sim H\sqrt{\varphi_0}$, i.e., for fixed H the quantity β is uniquely connected with the flow rate (pressure) of the neutrals, while the criterion $\varkappa$ for φ_0 = const is proportional to the magnitude of the magnetic field. An estimate of the magnitudes of the criteria for argon for $\langle\sigma_{ie}v_e\rangle = 1.5\cdot10^{-7}$ cm^2, $v_0 = 3\cdot10^4$ cm/sec, and $\nu_0/\nu_i = 3$ gives $\varkappa \simeq 2.2\cdot10^{-2}H\sqrt{\varphi_0}$, $\beta = 2.8\times10^{-15}n_n/H$, where H, kOe; φ_0, kV; n_n cm^{-3}.

Boundary-value problem (3), (4) was solved numerically by the method of successive approximations. Figures 2-5 show typical distributions over the sheath of the main discharge parameters (the numbers appended to the curves are the values of β). Figure 2 shows plots of the potential $\eta(s)$ and electron energy $\Theta(s)$; Fig. 3 shows the distributions of electron $j_e(s)$ and ion $j_i(s)$ current densities (solid and dashed curves, respectively). The curves shown are for k = 63, $\mathscr{E}$ = 0.05 (this corresponds, for example, to φ_0 = 1 kV and $\mathscr{E}_i \simeq 3$ eV$_i$ for argon), $\Theta_0 \simeq 3\cdot10^{-3}$, $\varkappa$ = 0.2. The coordinate s in Figs. 2 and 3 is reckoned from the anode; the curves are cut off, only the main region of variation of the corresponding functions being shown. Figure 4 shows the distributions of n_e and n_i (solid and dashed curves, respectively), while Fig. 5 shows the variation of the flux (density) of neutrals q. (To facilitate comparison with [5] the results presented in Figs. 4 and 5 are given as functions of $\sqrt{\eta}$.)

An analysis of the obtained results shows that the characteristics of the sheath are significantly affected by the weak processes over quite a wide range of criteria β and $\varkappa$. (This is illustrated by the corresponding dashed-dot curves shown for comparison in Figs. 2-5 for β = 0.5 and obtained in [5] ignoring weak processes.) As in [3], the distributions of all quantities are characterized by the formation of a "tail" at the beginning of

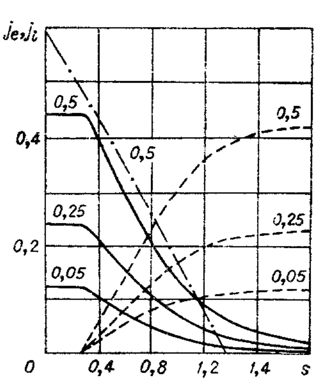

Fig. 3

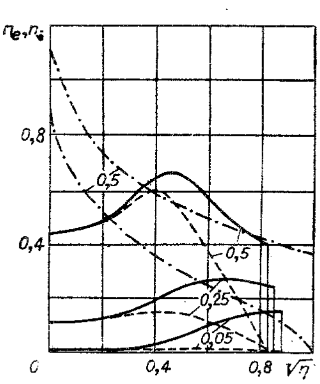

Fig. 4

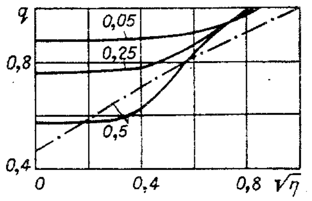

Fig. 5

the sheath, i.e., a region in which the corresponding functions vary insignificantly and where the conditions necessary for their subsequent rapid growth are formed. This region adjoins the part of the sheath in which almost all the ionization of the neutrals takes place and in which the main variation of electron energy, density, and flux occurs (and which, therefore, is naturally designated as the actual width of the sheath). The formal width of the sheath s_A may reach values 5-10, although its actual value proves to be for all cases at the level 2-3.

The distributions $j(s)$, $E(s)$ and $q(\sqrt{\eta})$ exhibit horizontal portions corresponding to the detachment region. The distribution $\eta(s)$ become slightly more sloping. The electric field E and the discharge current j_D are decreased. If, in the vacuum mode, the effect of weak processes is to reduce E_A and j_D by a factor of 2-3, then with increasing pressure to the level $\beta = 0.5$ this difference diminishes to $\sim 25\%$ and is determined primarily by the presence of the detachment region, whereas the effect of $\nu_i(W)$ and $\mathscr{E}$ becomes weak. The form of the density distributions n_e and n_i changes qualitatively and quantitatively (Fig. 4).* Instead of profiles that fall for $\beta < 1$ we obtain more physical distributions that are nonmonotonic for any β, including small β. At the beginning of the sheath a region of quasineutrality appears and the density n(0) is greatly decreased (for $\beta = 0.1$ by more than an order; for $\beta = 0.5$ by a factor of around two). The form of the distribution $q(\sqrt{\eta})$ changes, both near the anode (effect of detachment region) and at the beginning of the sheath (see Fig. 5). The weak processes decrease the probability of ionization of a neutral in the sheath $P = 1 - q(0)$, the effect of these processes decreasing with increasing β except in the detachment region. For example, for $\beta = 0.1$, the probability P is almost halved, while for $\beta = 0.5$ it is decreased by almost 25%. From the law of mass conservation we have the relationship $j_D = 1 - q(0)$ or $P = j_D$.† It follows from this that the presence of the detachment region reduces the probability of ionization of a neutral in the sheath by around 25%. The physical explanation of this is that the power $(\varphi_0 - \varphi_d) j_D$ released in the electron gas in the detachment region and also the energy flux carried by electrons from the diffusion part of the sheath cannot be utilized for the generation of particles. Despite the fact that the thickness of the detachment region is only around 10% of the actual thickness of the sheath, the drop across it for $\varkappa = 0.2$ and $\beta \leq 0.5$ is $(0.2\text{-}0.3)\varphi_0$ and the electrons gather around one-half of the energy carried to the anode ($\Theta_d = 0.2\text{-}0.31$, $\Theta_A = 0.41\text{-}0.61$). With increasing β and decreasing $\varkappa$ this effect becomes still more pronounced, since then $(\varphi_0 - \varphi_d)$ and Θ_A increase monotonically and the maximum value of the energy gathered by electrons in the diffusion part of the sheath Θ_d does not exceed 0.3.

*The distributions n_e and n_i shown by the dashed-dot curves in Fig. 4 are taken from [5]. The respective distributions obtained utilizing the condition of quasineutrality at the beginning of the sheath differ insignificantly from the cited distributions only in a small region near $\eta = 0$.

†In papers on plasma accelerators one encounters the working substance utilization coefficient, defined as the ratio of the total ion current to the flow rate [4]. Evidently, in a one-dimensional theory this coefficient equals P.

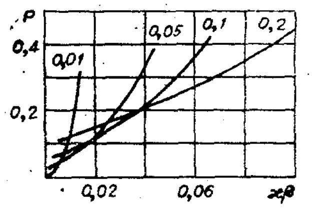

Fig. 6

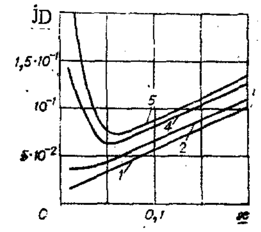

Fig. 7

For $\varkappa = 0.01$ and $\beta = 1.2$ the fall is already $0.53\varphi_0$ and the electrons gather two-thirds of the energy carried to the anode. The detachment region must therefore be taken into account in the discussion of a number of questions (e.g., estimation of energy dissipated at anode, determination of effective accelerating voltage for ions in the sheath, estimation of discharge current or ionization probability).

The distributions obtained for small β ($\beta \ll 1$) give an idea of the level of the quantities in a low-pressure discharge [3]. With increasing β the density distributions n_e and n_i and the fluxes q, j_e, and j_i change most strongly. There is an increase in the burning of neutrals with corresponding increases in the charged-particle density, electric field, and discharge and ion currents. For example, increasing β by an order (from 0.05 to 0.5) for $\varkappa = 0.2$ increases the density n_e by a factor of 4 and n_i by a factor of 30; the discharge current is thereby increased by a factor of 35† and the electric field only by 40%. At not too small β ($\beta \geq 0.1$) there is a dependence of the charged-particle density on pressure (see Fig. 4), in contrast to the vacuum mode [3]. The total space charge at the anode varies with increasing β. The space charge in the sheath increases at small flow rates, and as a result the electric field at the anode grows and the potential is developed over a smaller length (the sheath thickness decreases slightly). Further increasing of the flow rate leads to an increase in the thickness of the quasineutral part of the sheath and to the compression of the space-charge region towards the anode. The magnitude of the total space charge is thereby decreased and the growth of the electric field at the anode and the diminution of the sheath thickness cease. Figure 6 shows the variation of the probability P of ionization of a neutral in the sheath (or, what is the same thing, the dimensionless discharge current j_D) with increasing flow rate $\varkappa\beta$ for various values of the parameter $\varkappa$ (the numbers appended to the curves). The curves can be approximated by the expression

$$P(\varkappa\beta) = P_0(\varkappa) + P_1(\varkappa)\varkappa\beta + P_2(\varkappa)(\varkappa\beta)^2 + \ldots, \qquad (6)$$

where $P_0(\varkappa)$ is proportional to $\varkappa$ and the coefficients $P_1(\varkappa)$, $P_2(\varkappa)$, and so on, by contrast, decrease with increasing $\varkappa$ for $\beta \to 0$, $P \to P_0(\varkappa)$. This means that at sufficiently small flow rates the discharge current depends almost linearly on pressure, which is characteristic for the vacuum discharge mode. With increasing flow rate a role begins to be played by successive terms of expansion (6), corresponding to quadratic, cubic, etc., dependences of the discharge current on pressure. Under such conditions the smaller the magnetic field, the earlier the deviation from linearity begins and the more pronounced it is. Simultaneously with the deviation of the function "discharge current versus pressure" from linear, there appears a corresponding dependence of the Hall current on pressure. If curves are plotted of the dimensional discharge current $\varkappa\beta j_D$ versus the flow rate $\varkappa\beta$ (analogous to Fig. 6), it will be seen that, starting from definite characteristic values of the flow rates, the rate of growth of the current increases sharply with a certain accompanying increase in the thickness of the sheath. The values of these characteristic flow rates increase with increasing magnetic field. This phenomenon probably corresponds to the approach of the flow rate to the critical value at which the discharge goes over abruptly into the heavy-current mode. Numerical estimates and a comparison with experiment are in accordance with this hypothesis. By [4], for H = 1 kOe, $\varphi_0 = 400$ V, and bismuth as the working substance ($V_i = 7.3$ V, k = 55), the first measured point of the heavy-current mode corresponds to a flow-rate density $q_* \simeq 0.35$ A/cm^2. For these conditions, setting $\nu_0/\nu_i = 3$, $v_0 = 3 \cdot 10^4$ cm/sec, and $\sigma_{i0} = 4 \cdot 10^{-15}$ cm^2, we find $\varkappa \simeq 0.04$, $\beta \simeq 3.5\, q_A$. A rough estimate is obtained by utilizing the curves shown in Fig. 6 (for k = 63 and $\mathscr{E} = 0.05$). The abscissa of the last point of the curve $\varkappa = 0.05$ equals $\varkappa\beta = 0.045$, whence $\beta_* = 0.9$ and for q_* we find $q_* \simeq 0.26$ A/cm^2. The above-noted relationship between the critical flow rate and magnetic field is consistent with the condition for ionization instability [9]. The pairs of values $(\varkappa, \varkappa\beta)$ corresponding to the ends of the curves in Fig. 6 approximately define the transition boundary and probably give a generalization of the criterion of ionization instability with weak processes taken into account.

Let us consider, finally, the dependence of the discharge parameters on magnetic field. If Θ_d is effectively independent of magnetic field, the characteristics like charged-particle density, sheath thickness, and

†When comparing the discharge currents for different values of the parameters β and $\varkappa$ one must remember that the scale of j_D varies in proportion to $\varkappa\beta$.

discharge current density vary significantly with increasing $\varkappa$. For example, increasing $\varkappa$ from 0.05 to 0.2 (for a fixed flow rate $\varkappa\beta = 0.005$) decreases the sheath thickness by a factor of around 3, increases the discharge current density by a factor of around 3, and increases the electron density by a factor of around 10. As in the case of the vacuum mode of discharge, the sheath thickness is approximately inversely proportional to $\varkappa$ over the entire considered range of flow rates. The dependence $n_e(\varkappa)$ varies for different flow rates. For sufficiently small flow rates and any field an approximately quadratic dependence is observed: $n_e \sim \varkappa^2$ (as in the vacuum mode). With increasing flow rate this dependence is retained only for large magnetic fields ($\varkappa \geqslant 0.05$). If $\varkappa \leqslant 0.05$, a minimum appears in the function $n_e(\varkappa)$ and an interval on which it decreases. Figure 7 shows plots of $j_D(\varkappa)$ for a number of fixed values of the flow rate $\varkappa\beta$ = const (the numbers appended to the curves denote the flow rate in relative units). For small flow rates (curve 1) the plot of the function $j_D(\varkappa)$ is a straight line, which corresponds to the linear dependence of the discharge current on the magnetic field characteristic of the vacuum mode [2]. With increasing flow rate a linear dependence is maintained, just as for $n_e(\varkappa)$, only if the magnetic field is large enough. In the region of small magnetic fields ($\varkappa \lesssim 0.05$) increasing the flow rate, first of all, slows down the rate of growth of $j_D(\varkappa)$ (curve 2) and then leads to the appearance in the plot of the function $j_D(\varkappa)$ of a minimum and a region on which the function decreases (curves 4 and 5). So long as the electron density is proportional to H^2 and the sheath thickness is inversely proportional to H, the total number of electrons in the sheath and thus also the ionization probability or discharge current density are linearly dependent on the magnetic field. The decrease of $n_e(\varkappa)$ with increasing magnetic field obtained at large flow rates ($\varkappa\beta \gtrsim 0.01$) leads to a reduction of the total number of electrons in the sheath, which explains the behavior of $j_D(\varkappa)$ for small magnetic fields. We note that a falling dependence of the discharge current (or the working substance utilization coefficient) on magnetic field was observed in [4] and is characteristic for the heavy-current mode.

In this manner, the characteristics of the considered discharge mode have features associated with the vacuum and the heavy-current modes, although the properties of the latter mode are manifested to a lesser extent as the transition to it is not continuous. It also follows from the above results that weak processes have a significant effect on the structure of the anode sheath and on the characteristics of the transition discharge mode. The results on the heating of electrons and on the sheath thickness give a basis for a diffusion description of the discharge.

LITERATURE CITED

1. S. D. Grishin, V. S. Erofeev, and A. V. Zharinov, "Closed Hall current accelerators," in: Plasma Accelerators [in Russian], Izd. Mashinostroenie, Moscow (1973), p. 54; A. I. Morozov, ibid., p. 5.
2. N. A. Kervalishvili and A. V. Zharinov, "Characteristics of low-pressure discharge in a transverse magnetic field," Zh. Tekh. Fiz., 35, No. 12 (1965).
3. V. K. Kalashnikov and Yu. V. Sanochkin, "Theory of low-pressure self-sustaining discharge with closed electron drift," Zh. Tekh. Fiz., 44, No. 12 (1974).
4. V. S. Erofeev and E. A. Lyapin, "Integrated characteristics of Hall accelerator ion source with anode sheath," in: Materials of Second All-Union Conference on Plasma Accelerators [in Russian], Izd. Inst. Fiz. Akad. Nauk BelorusSSR, Minsk (1973), p. 130.
5. V. S. Erofeev, Yu. V. Sanochkin, and S. S. Filippov, "Near-anode electrical sheath in discharge with closed Hall current," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1969).
6. Yu. S. Popov, "Anode sheath in strong transverse magnetic field," Zh. Tekh. Fiz., 40, No. 8 (1970).
7. D. Bohm, "Minimum ion kinetic energy for a stable sheath," in: The Characteristics of Electrical Discharges in Magnetic Fields (edited by A. Guthrie and R. K. Wakerling), McGraw-Hill, New York–Toronto–London (1949).
8. N. N. Lebedev, Special Functions and Their Applications [in Russian], Fizmatgiz, Moscow (1963).
9. V. S. Erofeev and Yu. V. Sanochkin, "Ionization instability of low-pressure self-sustaining discharge in a strong transverse magnetic field," Zh. Tekh. Fiz., 40, No. 9 (1970).
10. N. A. Kervalishvili and V. P. Kortkhondiya, "Mechanism of low-pressure discharge in a transverse magnetic field," Zh. Tekh. Fiz., 43, No. 9 (1973).